

## Dam-reservoir interaction during earthquake

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### ABSTRACT

A novel technique developed for the finite element modelling of reservoir vibration is implemented in the earthquake response analysis of gravity dams to include the effects of energy dissipations due to the radiation and absorption of pressure waves in the reservoir. When the proposed radiation condition is used to model an infinitely long reservoir as a finite one of relatively very short length, some minor discrepancies are observed in the results for the complex frequency response of hydrodynamic pressures at frequencies near the second cut-off frequency of the reservoir vibration. The effect of such discrepancies on the earthquake response of dams is found to be negligible. Example analyses are conducted by using a past earthquake data to demonstrate the effectiveness of the proposed technique and to study the effects of hydrodynamic interaction on the earthquake response of dams.

### INTRODUCTION

The response of a dam to an earthquake depends very significantly on the effects of dam-reservoir interaction, compressibility of water in the reservoir and the absorption of pressure waves at the bottom of the reservoir (Chopra 1970; Chakrabarti and Chopra 1973; Fenves and Chopra 1983; Chopra 1987). In order to include these effects in the analysis of a dam having an arbitrary geometry, the complete dam-reservoir system must be discretized (Sharan and Gladwell 1985a). A computational difficulty arises in such a discretization because in most of the practical situations, reservoirs are infinitely long. Even if a relatively very large length of the reservoir is discretized, use of the conventional Sommerfeld radiation condition (Zienkiewicz and Newton 1969) does not produce satisfactory results (Humar and Roufaiel 1983). This difficulty may be circumvented by coupling the finite element model with continuum solutions (Hall and Chopra 1982) or boundary elements (Humar and Jablonski

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1989). Recently, a highly effective, very efficient and simple radiation condition was developed (Sharan 1985b, 1987) for the finite modelling of infinite reservoirs having a rigid bottom. The technique was then extended to include the effects of the deformability of reservoir bottom (Sharan, to be published). In these studies, the dam was idealized as being rigid or deformable plates and the analyses were limited to the complex frequency response of hydrodynamic pressures. The objective of this paper is to implement the technique in the earthquake response analysis of two-dimensional concrete gravity dams impounding reservoirs having a deformable bottom. The analysis is conducted in the frequency domain and the response of dams to an earthquake is obtained by using the fast Fourier transform.

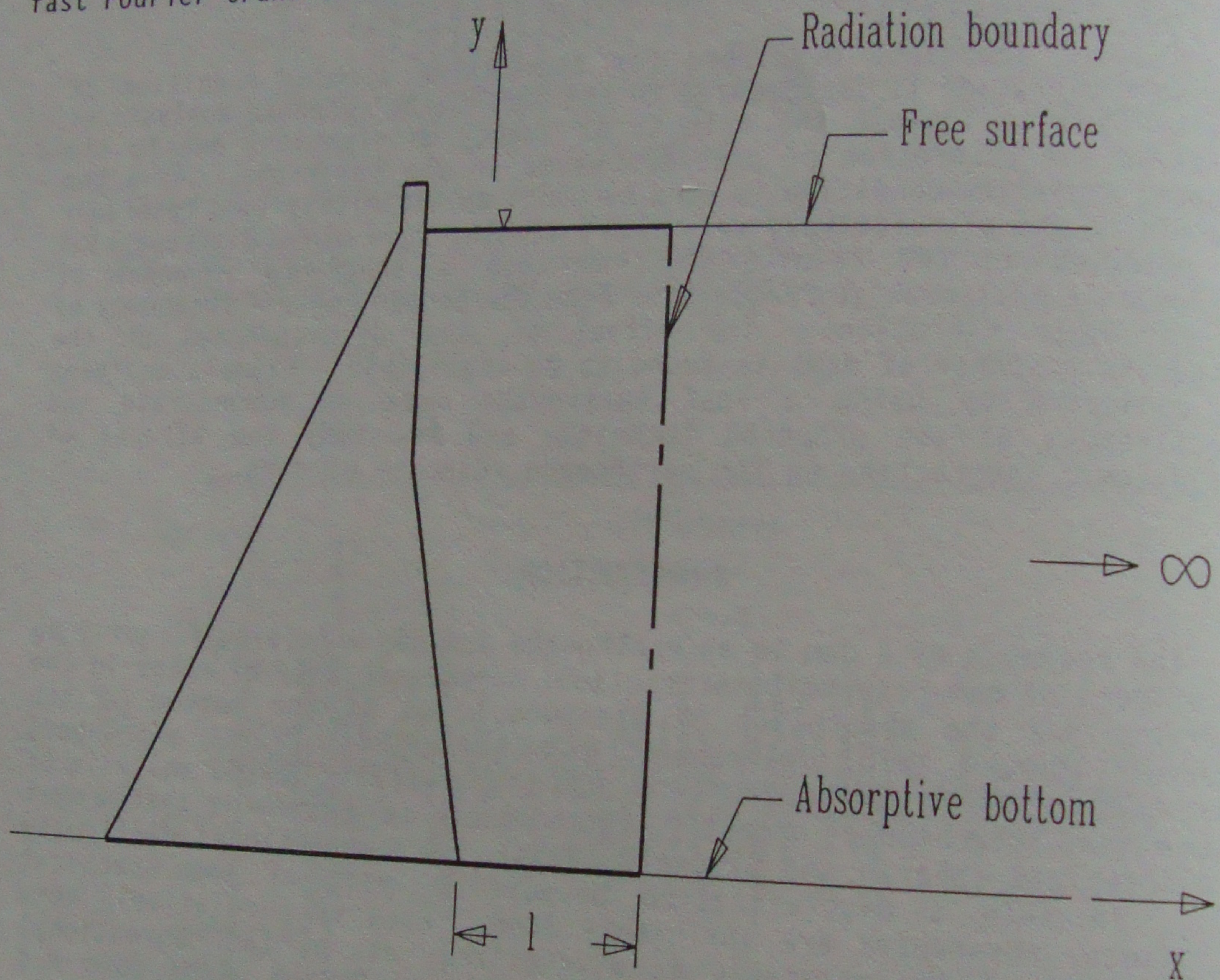


Figure 1. A dam-reservoir system

### FINITE ELEMENT ANALYSIS

By considering displacements and pressures to be the basic nodal unknowns for the dam and the reservoir, respectively, the finite element discretization (Zienkiewicz and Newton 1969) of a dam-reservoir system (Fig. 1) leads to the following coupled equations for the complex frequency response (Sharan 1987):



$$[A_{dr}]\{\bar{\delta}\} = -[M_{dr}]\{\bar{a}_g\} \quad (1)$$

$$[B_r]\{\bar{p}\} = -\rho [S_{dr}]^T (\{\bar{a}_g\} - \omega^2\{\bar{\delta}\}) \quad (2)$$

$$[A_{dr}] = [K_d] - \omega^2 [M_{dr}] + i\omega [C_d] \quad (3)$$

$$[B_r] = [K_r] - \omega^2 [M_r] + [C_r] \quad (4)$$

$$[M_{dr}] = [M_d] + \rho [S_{dr}] [B_r]^{-1} [S_{dr}]^T \quad (5)$$

$$\{\bar{a}_g, \bar{\delta}, \bar{p}\} = \{a_g, \delta, p\} e^{i\omega t} \quad (6)$$

In the above equations,  $[K_d]$ ,  $[C_d]$  and  $[M_d]$  are the stiffness, damping and mass matrices for the dam and  $[K_r]$ ,  $[C_r]$  and  $[M_r]$  are the corresponding matrices for the reservoir;  $[S_{dr}]$  is the coupling matrix;  $\{a_g, \delta, p\}$  are the vectors of nodal ground accelerations, displacements and hydrodynamic pressures, respectively;  $\rho$  is the density of water in the reservoir;  $\omega$  is the circular frequency of vibration,  $t$  is the time variable and  $i = \sqrt{-1}$ .

#### Damping matrix for the reservoir

Energy dissipation in the reservoir is caused by (i) dam-reservoir interaction, (ii) radiation and (iii) absorption of pressure waves at the bottom of the reservoir. The first type of damping is modelled by dam-reservoir coupling and the remaining two types are modelled through the damping matrix  $[C_r]$  in Eq. 4. Implementation of the absorbing boundary condition (Fenves and Chopra 1983) and the proposed radiation condition (Sharan, to be published) for a horizontal ground motion leads to the following form of the damping matrix:

$$[C_r] = i\omega q [C_{ra}] + \zeta [C_{rc}] \quad (7)$$

$$q = \frac{(1 - \alpha_r)}{c(1 + \alpha_r)} \quad (8)$$

$$\zeta = \frac{1}{d} \sqrt{\beta - \left(\frac{\omega d}{c}\right)^2} \quad (9)$$



where  $[C_{ra}]$  and  $[C_{rt}]$  are damping matrices corresponding, respectively to the absorbing and truncation boundary,  $\alpha_r$  is the reflection coefficient of pressure waves,  $c$  is the velocity of acoustic waves in water,  $d$  is the depth of water at the truncation surface and  $\beta$  is the first root of the equation

$$e^{2i\beta} = \frac{\omega q d - \beta}{\omega q d + \beta} \quad (10)$$

#### PARAMETRIC STUDY

The effectiveness and efficiency of the proposed method was tested by analyzing two different dams having the following dimensions (Fig. 2): (a) the Pine Flt dam ( $h = 121.9$  m,  $d = 116.1$  m,  $b = 95.8$  and  $t = 9.8$  m) to represent a high dam and (b) the Angostura dam ( $h = 48.5$  m,  $d = 44.9$  m,  $b = 42.2$  m and  $t = 3.0$  m) to represent a low dam having an inclined upstream face. Three different values of  $\alpha_r$  ( $= 1.0, 0.75,$  and  $0.5$ ) and three different locations of the truncation boundary given by  $\ell/d$  ( $= 0.5, 1.0$  and  $2.0$ ) were considered. A detailed description of material and geometrical properties of the systems may be found elsewhere (Sharan, 1985a). Three-noded triangular finite elements were used for the discretization and Fig. 2 shows a typical finite element model.

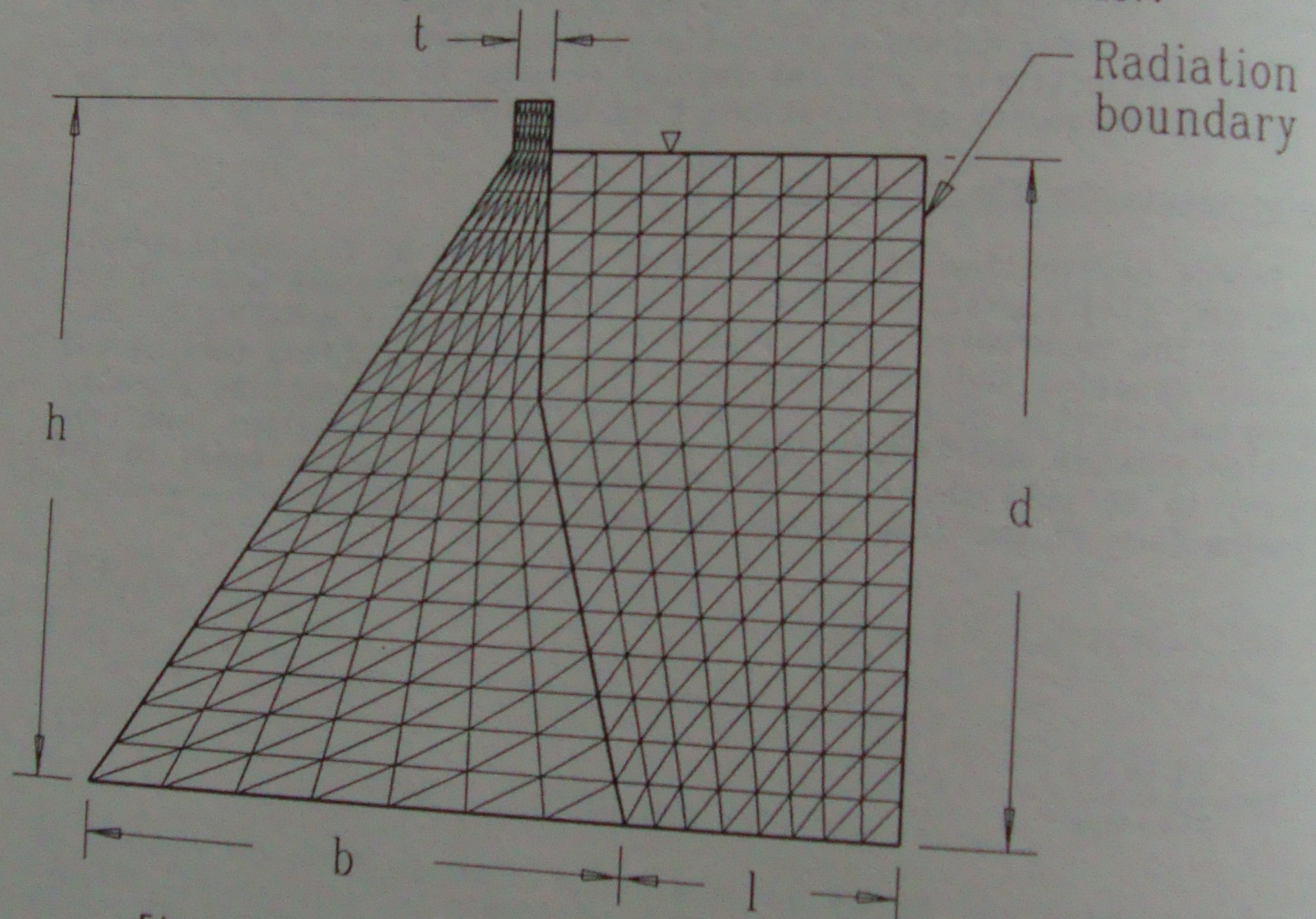


Figure 2. A typical finite element model of Angostura dam-reservoir system ( $\ell/d = 0.5$ )



Table 1. Complex frequency response of hydrodynamic forces on (a) Pine flat dam and (b) Angostura dam

Dam	$\omega/\omega_1$	$\alpha_r$	$ \bar{F}_{dyn} /F_{stat}$			
			Sommerfeld radiation		Proposed radiation	
			$\ell/d=0.5$	2.0	0.5	2.0
(a)	0	1.00	1.52	1.05	1.05	1.05
		0.75	1.52	1.05	1.05	1.05
		0.50	1.52	1.05	1.05	1.05
	1	1.00	2.88	11.95	5.86	5.90
		0.75	2.64	6.24	5.63	5.63
		0.50	2.42	4.26	4.10	4.10
	2	1.00	0.58	0.53	0.52	0.52
		0.75	0.55	0.54	0.51	0.51
		0.50	0.53	0.54	0.51	0.52
	3	1.00	0.43	0.51	0.43	0.51
		0.75	0.47	0.53	0.46	0.52
		0.50	0.50	0.52	0.49	0.51
4	1.00	0.18	0.20	0.18	0.20	
	0.75	0.18	0.19	0.18	0.19	
	0.50	0.18	0.18	0.17	0.18	
(b)	0	1.00	1.27	0.98	0.98	0.98
		0.75	1.27	0.98	0.98	0.98
		0.50	1.27	0.98	0.98	0.98
	1	1.00	2.37	6.44	29.60	30.42
		0.75	2.17	4.16	4.27	4.27
		0.50	1.97	3.00	2.95	2.96
	2	1.00	0.55	0.49	0.49	0.49
		0.75	0.54	0.51	0.49	0.49
		0.50	0.54	0.52	0.50	0.50
	3	1.00	0.72	0.80	0.73	0.81
		0.75	0.74	0.75	0.74	0.74
		0.50	0.75	0.73	0.74	0.72
4	1.00	0.09	0.10	0.10	0.10	
	0.75	0.10	0.12	0.10	0.12	
	0.50	0.10	0.14	0.11	0.14	



Table 2. Complex frequency response of horizontal displacements at the crest of (a) Pine Flat dam and (b) Angostura dam

Dam	$\omega/\omega_1$	$\alpha_r$	$ \bar{\delta}_{dyn} /\delta_{stat}$			
			Sommerfeld radiation		Proposed radiation	
			$l/d=0.5$	2.0	0.5	2.0
(a)	0	1.00	9.6	8.5	8.5	8.5
		0.75	9.6	8.5	8.5	8.5
		0.50	9.6	8.5	8.5	8.5
	1	1.00	33.8	94.7	28.4	28.7
		0.75	33.3	57.0	49.0	49.0
		0.50	33.0	46.0	44.3	44.3
	2	1.00	7.1	6.7	7.0	7.0
		0.75	7.4	7.0	7.2	7.2
		0.50	7.6	7.4	7.4	7.4
	3	1.00	5.0	4.9	5.0	4.9
		0.75	4.9	4.6	4.9	4.6
		0.50	4.7	4.7	4.8	4.8
4	1.00	1.2	1.2	1.2	1.2	
	0.75	1.3	1.4	1.3	1.4	
	0.50	1.4	1.5	1.4	1.5	
(b)	0	1.00	11.1	10.4	10.4	10.4
		0.75	11.1	10.4	10.4	10.4
		0.50	11.1	10.4	10.4	10.4
	1	1.00	32.3	60.5	173.0	178.8
		0.75	31.5	44.4	43.9	44.0
		0.50	30.8	37.7	37.0	37.2
	2	1.00	11.3	11.0	11.3	11.3
		0.75	11.6	11.2	11.4	11.4
		0.50	11.9	11.5	11.6	11.6
	3	1.00	7.0	5.9	6.9	5.8
		0.75	6.8	6.1	6.8	6.0
		0.50	6.7	6.6	6.8	6.6
4	1.00	2.1	2.1	2.1	2.1	
	0.75	2.2	2.2	2.2	2.2	
	0.50	2.2	2.3	2.2	2.3	



### Complex frequency response

Tables 1 and 2 show absolute values of the horizontal components of the total hydrodynamic force  $F_{dyn}$  (in excess of the hydrostatic force  $F_{stat}$ ) and the dynamic displacement  $\delta_{dyn}$  (in excess of the static displacement  $\delta_{stat}$ ) at the crest of dams subjected to a horizontal ground acceleration  $a_g = ge^{i\omega t}$ ,  $g$  being the acceleration of gravity. Results are presented for a few typical values of the frequency of excitation normalized with respect to the first cut-off frequency  $\omega_1$  ( $= 0.5 \pi c/d$ ) of the reservoir. The value of  $\omega = 3\omega_1$  corresponds to the second cut-off frequency of the reservoir. In these tables, a comparison is made between results obtained by using the proposed and the conventional Sommerfeld radiation conditions for two different locations of the truncation boundary. By imposing the proposed radiation condition, almost identical results were produced for  $l/d = 0.5, 1.0$  and  $2.0$ . However, some minor discrepancies were observed for frequencies near the second cut-off frequency of the reservoir. With the use of the conventional Sommerfeld radiation, no convergence could be achieved even for larger values of  $l/d$ , particularly for the case of  $\alpha_r = 1$  and  $\omega/\omega_1 = 1$  which is of greater importance in the earthquake response analysis.

### Earthquake response analysis

The south-component of El Centro, 1940 earthquake (Hudson and Brady 1971) was used for the analysis. The duration of earthquake considered was 10.24 sec and 8192 time increments were used for the fast Fourier transform. Results for the maximum crest displacement  $\delta_{max}$  (in excess of  $\delta_{stat}$ ) and its time of occurrence  $t_m$  were obtained by using the proposed technique and as shown in Table 3, results for  $l/d = 0.5$  and  $2.0$  were found to be almost identical for all the cases analyzed. The effect of  $\alpha_r$  was found to be very significant for the Pine Flat dam and almost insignificant for the Angostura dam.

Table 3. Response of (a) Pine Flat and (b) Angostura dams to El Centro earthquake obtained by using the proposed method

Dam	$\alpha_r$	$l/d$	$\delta_{max}/\delta_{stat}$ (at the crest)	$t_m$ (sec)
(a)	1.00	0.5	6.7	2.32
		2.0	6.7	2.32
	0.75	0.5	6.4	2.32
		2.0	6.4	2.32
(b)	1.00	0.5	6.1	2.64
		2.0	6.2	2.64
	0.75	0.5	6.0	2.48
		2.0	6.0	2.48



## CONCLUSIONS

A novel radiation condition is implemented in the finite element analysis of an infinite reservoir-dam system. The technique is found to be highly effective and very efficient. Although the effectiveness of the proposed radiation condition is slightly reduced for frequencies near the second cut-off frequency of the reservoir, its effect on the earthquake response of dams is found to be insignificant. Based on the limited analysis, it is concluded that the effect of absorption of pressure waves at the bottom of a reservoir may be very significant.

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